INQUIRE — LEARN TO THINK

Discrete Geometry

Student Handout · Grades 8–11 · 3–4 sessions

Group: _____ Date: _____

Part 1: The Question

Here is the question we will be exploring:

> ***In a world with exactly 6 points, can every straight line be bisected?***

What are your initial thoughts? What does this question mean? What would you need to know to answer it?

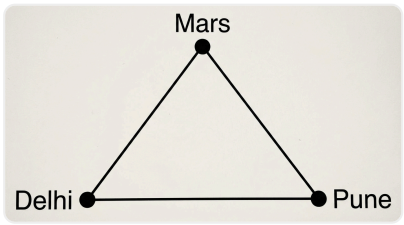
What words in the question need to be defined before you can answer it?

Part 2: Building the World

The teleportation analogy

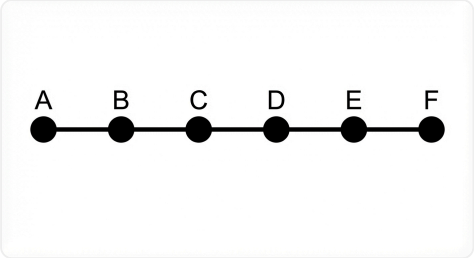
Imagine you are on Mars. There is a teleportation device that connects you to exactly two places: Delhi and Pune. From Delhi, you can teleport to Mars and Pune. From Pune, you can teleport to Mars and Delhi. There are no other ways to travel.

This is a world with 3 points and specific connections. Each dot is a point. Each line between dots is a connection (one hop). You cannot stop halfway along a connection — it is all or nothing, like teleportation.



The simple world

Here is a world with 6 points connected in a chain:



In this world, to get from A to D, you must go A → B → C → D. That is 3 hops, so the distance from A to D is 3.

Your own world

Draw a world with exactly 6 points. You can connect them however you like — but remember, each connection has the same length (1 hop).

DRAW YOUR WORLD HERE

How is your world different from the simple world?

Part 3: Straight Lines

In our discrete worlds, a **straight line** from point P to point Q is a **shortest path** from P to Q — the path that uses the fewest hops.

Finding straight lines in the simple world

Fill in the table. For each pair of points, write the shortest path and its length.

FROM	TO	SHORTEST PATH	LENGTH
A	B	A-B	1
A	C		
A	D		
A	E		
A	F		
B	C		
B	D		
B	E		
B	F		
C	D		
C	E		
C	F		
D	E		
D	F		
E	F		

How many straight lines are there in total? _____

What is the longest straight line? _____ What is the shortest? _____

Part 4: Bisection

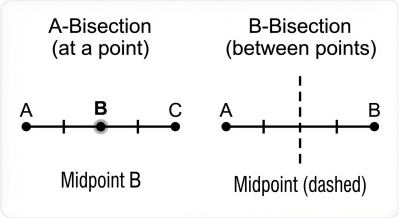
In Euclidean geometry, bisecting a line means dividing it into two equal parts. In a discrete world, there are two ways to interpret this:

A-Bisection (at a point): A straight line can be A-bisected if there is a point on the line that is equally far from both endpoints.

Example: The line A-B-C (length 2) can be A-bisected at B, because B is 1 hop from A and 1 hop from C.

B-Bisection (between points): A straight line can be B-bisected if it can be split into two equal halves, even if the split happens between two points (along an edge).

Example: The line A-B (length 1) can be B-bisected by splitting the edge A-B into two halves.



A-Bisection in the simple world

For each straight line, can it be A-bisected? If yes, which point bisects it?

STRAIGHT LINE	LENGTH	CAN IT BE A-BISECTED?	BISECTING POINT
A-B	1		
A-B-C	2		
A-B-C-D	3		
A-B-C-D-E	4		
A-B-C-D-E-F	5		
B-C	1		
B-C-D	2		
B-C-D-E	3		
B-C-D-E-F	4		
C-D	1		
C-D-E	2		
C-D-E-F	3		
D-E	1		
D-E-F	2		
E-F	1		

What pattern do you notice? Which lines can be A-bisected and which cannot?

Can every straight line in the simple world be A-bisected? _____

Challenge

Can you design a 6-point world where **every** straight line can be A-bisected?

DRAW YOUR WORLD HERE

Explain why every line in your world can be A-bisected:

Part 5: Circles

In Euclidean geometry, a **circle** is the set of all points at a fixed distance (the radius) from a centre point.

We use the same definition in our discrete worlds: a circle with centre P and radius r is the set of all points at distance r from P.

Circles in the simple world

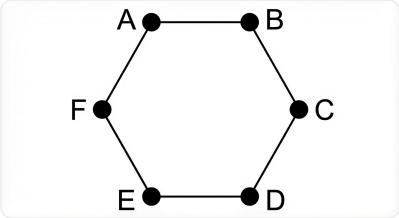
Fill in the table. For each centre and radius, list the points on the circle.

CENTRE	RADIUS	POINTS ON THE CIRCLE	NUMBER OF POINTS
A	1	B	1
A	2		
A	3		
A	4		
A	5		
C	1		
C	2		
C	3		
D	1		
D	2		
D	3		

What do you notice about the number of points on each circle?

Circles in the necklace world

The necklace world has 6 points connected in a cycle: A-B-C-D-E-F-A.



First, work out the distances. What is the distance from A to D? (Remember, distance is the length of the shortest path — and there may be more than one path.)

Distance from A to D: _____ (shortest path: _____)

Now fill in the table:

CENTRE	RADIUS	POINTS ON THE CIRCLE	NUMBER OF POINTS
A	1		
A	2		
A	3		
B	1		
B	2		
B	3		

How are these circles different from the ones in the simple world?

What is the maximum number of points on a circle in the necklace world? _____

Part 6: Triangles

In Euclidean geometry, a **triangle** is a closed shape with three straight-line sides.

In a discrete world, a triangle is three points P, Q, R with three straight lines (shortest paths): P to Q, Q to R, and R to P.

Two types of triangles

C-Triangle (collinear): All three points lie on a single straight line. Example: A, B, C in the simple world.

NC-Triangle (non-collinear): The three points do NOT all lie on a single straight line.

Triangles in the simple world

Pick any three points from the simple world (A-B-C-D-E-F chain). Are they a C-triangle or an NC-triangle?

Points chosen: _____, _____, _____

Side 1 (shortest path and length): _____

Side 2 (shortest path and length): _____

Side 3 (shortest path and length): _____

C-triangle or NC-triangle? _____

Can you find an NC-triangle in the simple world? Why or why not?

Triangles in the necklace world

In the necklace world (A-B-C-D-E-F-A), consider the points A, C, E.

Side 1 (A to C): shortest path = _____, length = _____

Side 2 (C to E): shortest path = _____, length = _____

Side 3 (E to A): shortest path = _____, length = _____

Is this a C-triangle or NC-triangle? _____

Is it equilateral (all sides equal)? _____

Can you find other NC-triangles in the necklace world? List them:

Part 7: Generalisation

Even versus odd necklace worlds

Think about necklace worlds with different numbers of points.

5-point necklace (A-B-C-D-E-A)

Maximum distance between any two points? _____

Can every straight line be A-bisected? _____

6-point necklace (A-B-C-D-E-F-A)

Maximum distance between any two points? _____

Can every straight line be A-bisected? _____

7-point necklace (A-B-C-D-E-F-G-A)

Maximum distance between any two points? _____

Can every straight line be A-bisected? _____

Do you notice a pattern? State your conjecture:

Open questions

Choose one of these to investigate further:

1. Can you design a 6-point world where every triple of points forms an NC-triangle?
2. What is the smallest world that has exactly one NC-triangle?
3. In a world with n points, what is the maximum number of equilateral triangles?
4. Can you find a world where a circle has more than 2 points?

Your chosen question: _____

Your investigation:

Part 8: Reflection

1. *What surprised you most about geometry in finite-point worlds?*

2. *Which was the hardest concept to extend from Euclidean geometry to discrete worlds — straight lines, bisection, circles, or triangles? Why?*

3. *Did you and your group ever disagree about a definition? What happened?*

4. *If you could explore any question about discrete worlds further, what would it be?*